Mathematical analysis of

Bluff

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Introduction

Bluff is a new Player vs. Dealer casino table game, played at a Blackjack-style gaming table and dealt from a shoe containing 2 to 8 standard 52-card decks. Each player plays a game of One Card Stud Poker with the dealer, with one raise equal to the ante allowed. Ace is high, Deuce is low. Side bets based on the player's card matching the dealer's card are available.

Game Rules

We omit specifics of the table layout and game procedures and focus on the math from the point of view of one player. The player begins by making an Ante wager. One card is dealt to the player, and he may then decide either to Check, leaving his Ante wager on the table, or Raise by the Ante amount, thus doubling his overall wager.

The dealer receives one card which is seen by the dealer or a card reader but not by the player. If the player has Raised, the dealer must Call or Fold. If the player has Checked, the dealer may either Check, or Raise by the Ante amount. If the dealer Raises, then the player must Call or Fold.

If the player Raises and the dealer Folds, the player is paid even money on his Ante wager, but his Raise wager is pushed. If the player Checked and the dealer then Raised and the player Folds, the player loses his Ante wager. Otherwise, there is a showdown and cards are compared. If the player has a lower card than the dealer, he loses his Ante bet, as well as, if made, his Raise or Call bet. If the player has a higher card than the dealer, he is paid even money on the Ante bet and, if made, his Raise or Call bet. If the player and dealer cards have the same rank (suits ignored), these bets all push.

The dealer follows a fixed strategy based on his card. If the player Raised, the dealer Folds with 2, 3, 4, 5, or 6, and otherwise Calls. If the player Checked, the dealer Checks with 4, 5, 6, 7, or 8, and otherwise Raises.

Independently, before any cards are dealt the player may make on optional side bet known as "Suit and Tie", and an optional $1 Progressive side bet. The Suit and Tie side bet pays according to a pay table if the player's card and the dealer's card have the same suit or rank (or both, which is possible because the game is played from a multi-deck shoe). The Progressive side bet extends over 2 or 3 consecutive deals, and pays only if the player's card matches the dealer's card in both suit and rank, according to a pay table.

We will first analyze the math for 2, 4, 6, or 8 decks, assuming the player does not see any cards other than his own and does not count cards. Then we will analyze the reduction in House Edge that can be obtained by using information on deck composition, either because the game is being dealt face up and the cards of other players can be seen, or because the player uses card counting to improve his play.

Basic Math Analysis

The player has three possible strategies for each of the thirteen possible card ranks he holds: Raise, Check and Fold if Raised, and Check and Call if Raised. The result of each strategy summed over all remaining cards in the shoe was calculated. The 6-deck calculation is illustrated here:

Decks	6	Dealer Folds 2-6, Checks 4-8				
House Edge		**4.230%**				
		Bet	Chk/Fold	Chk/Call	Optimal	Dealer
2	24	-265	-311	-456	-265	Fold/Bet
3	24	-265	-311	-360	-265	Fold/Bet
4	24	-265	-288	-288	-265	Fold/Check
5	24	-265	-240	-240	-240	Fold/Check
6	24	-265	-192	-192	-192	Fold/Check
7	24	-216	-144	-144	-144	Call/Check
8	24	-120	-96	-96	-96	Call/Check
9	24	-24	-71	-24	-24	Call/Bet
T	24	72	-71	72	72	Call/Bet
J	24	168	-71	168	168	Call/Bet
Q	24	264	-71	264	264	Call/Bet
K	24	360	-71	360	360	Call/Bet
A	24	456	-71	456	456	Call/Bet

Player	Dealer	Player	P Hi Pay	Tie Pay	P Lo Pay
Bet	Call	n/a	2	0	-2
Bet	Fold	n/a	1	1	1
Check	Bet	Call	2	0	-2
Check	Bet	Fold	-1	-1	-1
Check	Check	n/a	1	0	-1

The optimal player strategy is to Raise (bluffing) with 2, 3, and 4, and to Check with 5, 6, 7, and 8 (if Raised after Checking, Calling and Folding are equally good). With 9 and above, the player does equally well with Raising, or with Checking and Calling if Raised.

The House Edge of 4.230% is obtained as the average of the last column divided by 311 (the number of remaining cards available to be the dealer card).

With 2, 4, and 8 decks the House Edge is, respectively, 4.406%, 4.274%, and 4.208%. In the limit as the number of decks goes to infinity, the House Edge is 7/169 or 4.142%.

The side bets were analyzed for several pay tables. The 6-deck results are shown on the next page:

Suit and Tie			"To 1" pays	
		PT1	PT2	PT3
S&T	0.01607717	15	9	5
Rank	0.057877814	3	5	2
Suit	0.231511254	1	1	2
Neither	0.694533762	-1	-1	-1
	House Edge	**4.823%**	**2.894%**	**3.537%**

Progressive 2			"For 1 " pays			
		PT1	PT2	PT3	PT4	PT5
2 match (seed)	0.000258806	2000	2000	2000	1500	1500
1 match	0.015818365	25	20	20	15	15
No match	0.98392283	0	0	0	0	0
	Contrib	0.00%	0.00%	10.00%	25.00%	20.00%
	House Edge	**8.69%**	**16.60%**	**6.60%**	**12.45%**	**17.45%**
	Avg Jackpot	2000.00	2000.00	2386.39	2465.98	2272.78
Jackpot 1 in	3863.902724					

Progressive 3			"For 1 " pays			
		PT1	PT2	PT3	PT4	PT5
3 match (seed)	4.17145E-06	50000	50000	30000	30000	10000
2 match	0.000254634	1000	1000	1000	1000	1500
1 match	0.015818365	25	20	15	15	15
No match	0.98392283	0	0	0	0	0
	Contrib	0.00%	10.00%	25.00%	20.00%	20.00%
	House Edge	**14.13%**	**12.04%**	**13.29%**	**18.29%**	**13.91%**
	Avg Jackpot	50000.00	73972.46	89931.15	77944.92	57944.92
Jackpot 1 in	239724.581					

The probabilities were calculated using exact formulas, assuming that no shuffling takes place between successive hands for the Progressive. The progressive pay tables are only intended for use with a 6-deck shoe, because the House Edge is very sensitive to the number of decks. For the "Suit and Tie" side bet, the House Edge with 2, 4, and 8 decks is as follows:

2 decks – PT1 14.563%, PT2 8.738%, PT3 6.796%

4 decks – PT1 7.246%, PT2 4.348%, PT3 4.348%

8 decks – PT1 3.614%, PT2 2.169%, PT3 3.133%

Advanced Math: Card Counting Analysis

Because the player's optimal strategy is precisely balanced between calling and folding when holding a 5 through 8, and precisely balanced between raising and checking when holding a 9 through Ace, almost any knowledge about the composition of the remaining unseen cards is useful.

1) In particular, if the number of 9-A cards left exceeds or falls short of three times the number of 2-3 cards left, the player should fold or call, respectively, when raised after checking with 5-8.
2) Also, if the number of 7-8 cards left exceeds or falls short of the number of 2-3 cards left, the player should raise or check, respectively, on 9-A (and call if raised).
3) Although the previous 2 conditions can apply immediately, there is another condition which may arise later in the shoe: if the number of 7-A cards left is at least twice the number of 2-6 cards left, the player should not bluff.
4) There are further changes to the optimal strategy when more complex conditions are met, which could recommend checking on 4, bluffing on 5, folding on 9, and even rarer situations, but they are both more difficult to play and have a tiny overall impact.

An analysis was done with a 6-deck shoe, assuming either that the player used the first 3 criteria above to inform his play, or that he played mathematically perfectly.

If the shoe is dealt down to ½ deck left out of 6, the House Edge drops from 4.23% to 2.38% using the 3 counts above, and to 2.28% with perfect play.

If the shoe is dealt down to 1 deck left out of 6, the House Edge drops from 4.23% to 2.74% using the 3 counts above, and to 2.68% with perfect play.

If the shoe is dealt down to 1½ decks left out of 6, the House Edge drops from 4.23% to 2.94% using the 3 counts above, and to 2.89% with perfect play.

In the first case, using counts 1), 2), and 3) reduces the House Edge by 1.01%, 0.68%, and 0.16% respectively, with an additional 0.10% obtainable from perfect play. Realistically, many players will be able to keep the first two counts but only an expert will bother with the third, without which the House Edge can still be reduced to 2.54%.

Although changing strategy in this way is simple and efficacious, it is much more difficult to gain an advantage by changing one's bet, because the overall game is rarely in the player's favor. For example, if the shoe is dealt down to ½ deck left out of 6, then the game will favor the player on only 12.5% of the hands, by an average of 2.20% when it does. A player who quintupled his bet on positive shoes would earn only 0.125 x 0.022 x 4 = 1.1% of the Ante amount from the bet increase averaged over all hands, not nearly enough to overcome the 2.28% House Edge for perfect play with no bet increases.

The side wagers are even less vulnerable to card counting or to computer-perfect betting.

One other type of "card counting", which does not require any mental effort or a good memory, occurs in the "face up" version of the game, when it is dealt so that the players can see the cards of the other players. The game is played at a table with spots for up to 7 players, so that up to 6 extra cards are visible. Using information from these 6 cards according to the counting principles above, one can reduce the "face up" House Edge as follows:

8 decks – from 4.21% to 4.05% of the Ante

6 decks – from 4.23% to 4.02%

4 decks – from 4.27% to 3.96%

2 decks – from 4.41% to 3.75%

One final point about House Edge: although it is expressed in terms of the Ante, it can also be expressed as a percentage of the total amount bet by the player, which averages between 1.51 and 1.88 Ante units depending on strategy. If the most aggressive basic strategy is adopted (check only on 5-8, call if raised), the House Edge as a percentage of the total bet is only 2.25% when using 6 decks.

Appendix: Math analysis for High Limit Bluff including Card Counting analysis:

This variation is played with a 6 deck shoe, which is shuffled with 1, 1.5, or 2 decks left.

The first bet after the ante is called a "raise".

The Player may reraise if the Dealer bets. All bets (raises, reraises, and calls) must equal the ante.

The Player goes first and checks or bets. If he checks, the Dealer checks with 45678, and bets with 239TJQKA.

The Dealer folds with 2345 and red 6s, calls on black 6s and 789TJQKA, and also folds 239 to a reraise.

Optimal strategy: Bet with 239TJQ, checK/Call or checK/Fold with 45678, checK/Reraise with KA.

House Edge: 74/4043 = **1.8083%.** Average bet 279/169=1.65 to 319/169=1.89 depending on fold policy.

In the table below, the column headers are cut card placement (cards left out of 6x52=312).

House and player advantages are as percent of Ante not percent of total bet including raises and calls.

The "bet edge" is the average player advantage when positive, times the probability it is positive.

The "bet opt" player edge assumes perfect play, multiplying the ante when the shoe is positive, and shows the average win per hand as a percentage of the average ante (not total amount bet).

F vs C count: 23 seen is -3, 9TJQKA seen is +1, fold 4-8 when negative, call 4-8 when positive.

B vs K count: 2-5, Red 6 are -2. Black 6, 7-A are +1. Start at +12, bet on 23 when positive, else check/fold.

It is not feasible for anyone but a very expert card counter to get the last 0.1-0.3% without computer aid, and that still doesn't beat the game. The only threat to the casino comes from players who vary their bets or enter late in the shoe, but estimating whether the player has an advantage is much more difficult than it is in Blackjack. The basic counts above used to guide play decisions don't suffice, and even with computer aid the advantage from bet variation is smaller than can be achieved in Blackjack.

Place cut card at	52	78	104
Basic Strategy House Edge	1.8%	1.8%	1.8%
Gain from 4-8 F vs C count	1.1%	1.0%	0.9%
Gain from 23 B vs K count	0.3%	0.2%	0.2%
Gain from other Reraises	0.2%	0.1%	0.1%
Miscellaneous non-basic plays	0.1%	0.1%	0.0%
House Edge with Perfect play	0.1%	0.4%	0.6%
Bet edge (positive shoes)	0.7%	0.5%	0.3%
Positive shoe frequency	31.8%	24.2%	17.4%
Player edge 3x bet opt	0.8%	0.4%	0.0%
Player edge 5x bet opt	1.2%	0.8%	0.4%
Player edge 10x bet opt	1.6%	1.2%	0.9%